Disappearance of relaxation oscillation frequencies in a multimode solid-state laser

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Abstract

We study analytically the Tang, Statz and deMars rate equations describing a solid-state Fabry-Perot laser. When the modes have equal gains, there is a critical number of lasing modes, above which the low-frequency relaxation oscillations responsible for antiphase dynamics disappear. These results are generalized to include unequal modal gains resulting from a Lorentzian gain profile. © 1997 Published by Elsevier Science B.V.

1. Introduction

The current renewed interest in the dynamics of multimode free running Fabry-Perot lasers is caused by the emergence of a new generation of compact solid-state lasers with diode laser pumping which possess previously inaccessible low noise level and high stability. Investigations of the dynamics of these lasers were focused on their relaxation and noise characteristics. It is well known that in single-mode lasers, relaxation to the steady state occurs via damped oscillations. These oscillations are found in the power spectrum of intensity fluctuations as a peak at the relaxation or McCumber frequency $\Omega_R$ [1]. In multimode solid state lasers, a more complex dynamics occurs. At $\Omega_R$, all modes and the total intensity display a peak in their power spectrum (inphased dynamics). However, the modal intensities also display peaks at a group of low-frequency relaxation oscillations ($\Omega_{\text{L},p} < \Omega_R$). In the total intensity, the peaks at these frequencies are much weaker, showing signs of compensated antiphased oscillations. This behavior has been reported experimentally in the transient relaxation of multimode lasers in Fabry-Perot configurations, in the response of such lasers to an external periodic modulation and in the noise spectrum of multimode lasers [2–4].

The common wisdom is that there are as many relaxation frequencies as there are modes. In fact this turns out to be only the upper bound. As we demonstrate in this Letter, the number of relaxation frequencies can take any value between zero and the number of modes in the framework of the Tang, Statz and deMars (TSD) rate equations [5]. In this model, the modal intensities are coupled to the population inversion which is approximated as the combination of a spatially uniform component and the amplitude of a grating at each spatial frequency of the standing wave pattern formed by an oscillating mode. Such models require that the intermode spacing be sufficiently larger than the cavity bandwidth. The purpose of this Letter is to present new results on the linear stability analysis of the TSD equations and investigate the dis-
appearance of the low relaxation frequencies at a large number of lasing modes.

Our analysis is an extension of the recent theoretical work on the spectral properties of the TSD equations \[2,3,6-8\] which we write as

\[\frac{dI(p,\tau)}{d\tau} = k\{\gamma_p[D(\tau) - D(p,\tau)] - 1\}I(p,\tau),\]

\[\frac{dD(\tau)}{d\tau} = w - \left(1 + \sum_p I(p,\tau)\gamma_p\right)D(\tau) + \sum_p I(p,\tau)\gamma_pD(p,\tau),\]

\[\frac{dD(p,\tau)}{d\tau} = -\left(1 + \sum_q I(q,\tau)\gamma_q\right)D(p,\tau) + \frac{1}{2}I(p,\tau)\gamma_pD(p,\tau) . \tag{1}\]

These equations describe the interaction of the \(N\) modal intensities \(I(p,\tau)\) with the \(N\) population gratings \(D(p,\tau)\) and the average population inversion \(D(\tau)\), which are defined as

\[D(\tau) = \frac{1}{L} \int_0^L \tilde{D}(z,\tau) \, dz,\]

\[D(p,\tau) = \frac{1}{L} \int_0^L \tilde{D}(z,\tau) \cos(2kpz) \, dz . \tag{2}\]

The active medium fills the whole cavity. Time is measured in units of the population inversion relaxation time. Given the scaling of the time, \(k\) is the photon decay rate divided by the population inversion decay rate. The parameter \(k\) is assumed to be mode independent, but the gain of mode \(p\) relative to the gain of the mode nearest to the line center, \(\gamma_p\), is mode dependent. Thus, the lasing threshold for the central mode is \(w_0 = 1\), the pump threshold of any other mode is \(w_p \geq 1\). The relative gain satisfies the condition \(\gamma_p \leq 1\).

The steady state solution of the TSD equations can be found for arbitrary relative gains [6],

\[D(p) = D - 1/\gamma_p, \quad w = D + \frac{DS_1 - S_2}{S_1 - (N - 1/2)D},\]

\[S_1 = \sum_p 1/\gamma_p, \quad S_2 = \sum_p (1/\gamma_p)^2 ,\]

\[I(p) = \frac{1}{\gamma_p} \frac{D - 1/\gamma_p}{S_1 - (N - 1/2)D} . \tag{3}\]

2. Equal modal gains

In Ref. [6] we have shown that a simple reference model can be set up in the limit \(k \to \infty\) and equal gains. With this reference model, it was proved that only two frequencies

\[\Omega_k^2 = [k(w - 1)]^{1/2}, \tag{4}\]

\[\Omega_k^2 = [kD(w - 1)/2N]^{1/2} \tag{5}\]

characterize the relaxation to steady state, irrespective of the mode number \(N\) provided it is larger than unity. The frequency \(\Omega_k^2\) is the usual single-mode laser relaxation frequency while the low frequency \(\Omega_{1L}\) is \(N - 1\) times degenerate. The main simplification due to the limit \(k \to \infty\) is that to dominant order the oscillations are not damped. In this Letter we extend this analysis to include the effect of the finite cavity decay rate \(k\). However, since solid-state lasers have cavity decay rates of the order of \(10^3\) to \(10^6\), the limit \(k \gg 1\) will be used when useful. We will also assume that the number of lasing modes is large,

\[N \gg 1 . \tag{6}\]

Under these assumptions the steady state solution (3) becomes

\[D = 1 + D(p) + O(1/N^2),\]

\[D(p) = \frac{w - 1}{2wN} + O(1/N^2) ,\]

\[T = \frac{w - 1}{N} + O(1/N^2) . \tag{7}\]

Using the TSD equations (1), a linear stability analysis of the degenerate (\(\gamma_p = 1\)) steady state solution (7) yields the characteristic equation

\[P(2, \lambda) = \lambda^2 + \lambda w + (\Omega_k^2)^2,\]

\[P(3, \lambda) = \lambda^3 + 2\lambda^2 w + \lambda k(w - 1) + kw(w - 1) . \tag{8}\]

The analysis that follows rests on the fact that the quadratic \(P(2, \lambda)\) has complex roots \(\lambda_L\) if and only if its discriminant is negative,

\[(w/2)^2 - (\Omega_k^2)^2 < 0 . \tag{9}\]
If we look at the expression (5) for $\Omega^0_L$ we see that increasing the number of lasing modes decreases the value of this frequency. For $N \geq N_{sup}$, where

$$N_{sup} \equiv 2k(w - 1)/w^2 + O(1), \quad (10)$$

the low relaxation frequency

$$\Omega_L = \text{Im}(\lambda_L) = \left[ (\Omega^0_L)^2 - (w/2)^2 \right]^{1/2} \quad (11)$$

will vanish (Fig. 1). The roots of the cubic equation

$$P(3, \lambda) = 0$$

are

$$\lambda_1 = -w + O(k^{-1/2}),$$

$$\lambda_R = -w/2 \pm i\Omega^0_R + O(k^{-1/2}).$$

They are independent of the number of lasing modes and for $N \gg N_{sup}$ there remains only the relaxation oscillation which is close to the usual McCumber frequency $\Omega_R \approx \Omega^0_R + O(1/k)$.

### 3. Lorentzian gain line

The results obtained so far rely on the assumption of equal modal gains and arbitrary mode number. In general, the number of modes is a function of the gain profile as well as of the pump parameter. Let us consider the case of a Lorentzian distribution of the modal gains with one mode tuned to line center,

$$\gamma_p = \frac{1}{1 + (p\Delta)^2}, \quad p = 0, \pm 1, \pm 2, \ldots \pm m, \quad (12)$$

where $\Delta = \delta f_m/\delta f_g$ is the ratio of the intermode spacing to the gain linewidth. Using the steady state solution (3) and assuming $N\Delta \ll 1$, we can estimate the number of lasing modes as [8]

$$N = \left[3(w - 1)/w^2 \right]^{1/3}. \quad (13)$$

$N$ increases rapidly with increasing $w$ near the oscillation threshold and reaches the limit $(3/\Delta^3)^{1/3}$ for $w \to \infty$. Hence, for large $w$ the total number of lasing modes is restricted only by the parameter $\Delta$. Using the expression (10) for the cut-off mode number, we can formulate the condition of disappearance of low-frequency relaxation oscillations $N > N_{sup}$ as

$$\Delta^2 < \frac{3w^5}{(2k)^3(w - 1)^2}. \quad (14)$$

It should be noticed that in the case of different modal gains $\gamma_p$ there are $p$ pairs of modes with equal gains. In spite of this degeneracy, the low frequencies $\Omega_{L,q}$ with $q = 1, \ldots, N - 1$ are different [6,9], unlike in the limit $k \to \infty$ [6]. Note that the use of (10) gives only a qualitative result. Therefore, if $N \approx N_{sup}$ it is possible that only part of these frequencies disappears. This case is shown in Fig. 2 for $w = 1.7$ and $\Delta^2 = 3 \times 10^{-5}$, where there are 35 lasing modes. However, we observe only 9 relaxation oscillations which coexist with 53 real roots. Decreasing the parameter $\Delta$ leads to an increase of $N$ and of the number of real roots until all complex conjugate roots disappear. An example of this situation is displayed in Fig. 3. So there are two control parameters, $w$ and $\Delta$, which govern the number of lasing modes as well as the number of relaxation oscillations frequencies.

Expression (14) can be reformulated as a condition for the cavity length

$$L > \frac{c}{2\delta f_g\Delta} > \frac{ck(w - 1)}{\delta f_gw^2} \sqrt{\frac{2k}{2w}}. \quad (15)$$

We therefore conclude that if $k$ is large, either a long cavity or a broad gain line are needed to fulfill the condition $N > N_{sup}$. For example, let us consider a Nd:YAG laser operating on the 1064 nm transition. In this case $\delta f_g = 2 \times 10^{11}$ Hz. For this parameter, $k = 10^2$ and $w = 2$, the expressions (15) lead to $L > 7$ m which is not a usual experimental condition.
The main result of this Letter is that in the framework of the TSD model, the number of low-frequency relaxation oscillations of a multimode laser is not a constant but can assume all values from zero to $N - 1$. The number of relaxation oscillations depends on all the laser parameters, viz. the number of modes, the pumping rate and the photon decay rate $k$. The low-frequency relaxation oscillations disappear whenever the relation (9) is not verified, such as in the case in the large mode limit (10).

If the shape of the gain line is taken into account, two groups of eigenvalues emerge. An example of this situation is displayed in Fig. 2 where there are 9 pairs of complex conjugate roots and 26 real roots for $w = 1.7$.

Experimental evidence of the disappearance of relaxation oscillations should be observable in Nd:YAG laser with long cavity. A more suitable candidate should be a Ti:sapphire laser, which combines a broad gain line with a relatively small value for $k$ (typically $10^2$ to $10^3$ compared to the usual range $10^4$ to $10^6$).

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### References
