Introduction to Superfluidity near localization: Supersolid and Superglass

Bởi:

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In this chapter, we briefly introduce the research theme of this thesis, which is superfluidity in physical settings where particles have the tendency to localize. Localization may arise from crystallization of the many-body system, occurring either spontaneously as a result of the interactions among elementary constituents, or induced by an external “pinning” potential or by confinement or disorder. We address several specific issues in this broad context by studying a simple model of lattice bosons, which allows one to answer a host of questions in a sufficiently general, unified language. The goal is not that of obtaining quantitative predictions for specific experimental systems (e.g., liquid helium), but rather that of gaining insight into the behaviour of superfluids near crystallization or in disorder.

The superfluid phenomenon

The year 1937 marked the discovery of the phenomenon of superfluidity (SF) which was first observed experimentally in bulk liquid $^4$He independently by Kapitza [link] and by Allen and Misener [link]. It was found that $^4$He undergoes a transition at the temperature $T_\lambda \approx 2.17K$ from a viscous fluid (referred to as a normal fluid) above $T_\lambda$ to a fluid with practically no viscosity (referred to as a superfluid), capable of sustaining dissipationless flow below $T_\lambda$.

Superfluidity is perhaps the most spectacular manifestation of quantum mechanics on a microscopic scale; as such, it is regarded as a low temperature phenomenon, i.e., it arises in a setting where the physical behaviour of the system is essentially dominated by its ground state properties. SF is related to Bose-Einstein condensation (BEC) phenomenon [link], [link], but the relationship is subtle, and in some regards still unclear [link]. For a translationally invariant system, BEC is equivalent to the occurrence of off-diagonal long range order (ODLRO). Bose statistics seems intimately connected to SF; indeed, the appearance of pairs of particles seems a necessary step in the stabilization of the
superfluid phase of fermi systems, as pairs acquire integer spin, and can therefore be regarded as behaving like bosons, under specific circumstances.

In the form of persistent flow, SF has so far been experimentally observed only in the two isotopes of helium (\(^{4}\)He and \(^{3}\)He), even though progress in the stabilization of assemblies of ultracold atoms may soon pave the way for the experimental study of SF in different, perhaps more controllable settings. The main obstacle to observing SF in other condensed matter systems that could potentially display it, such as molecular hydrogen, is the fact that at sufficiently low temperature crystallization occurs. As atoms or molecules become localized, transport is impeded and SF ceases to occur. Helium, on the other hand, under the pressure of its own vapour, remains a liquid all the way down to zero temperature (see, for instance, Ref. [link]).

In general, it seems natural to regard localization as an “enemy” of both BEC and SF, but an active line of theoretical and experimental investigation in low temperature physics aims at identifying specific physical systems and/or settings, in which SF and localization may coexist in a single, homogeneous phase of matter. The earliest proposal for this to occur was that of Andreev and Lifshitz, who, over four decades ago, speculated that a phase of matter known as supersolid, simultaneously displaying crystalline order (rigid, or diagonal long range order with broken translation symmetry) and superfluidity (superflow, and the concomitant off-diagonal long range order with broken \(U(1)\) gauge symmetry). Andreev and Lifshitz, as well as other authors [link], [link], proposed that solid helium could be a candidate for the observation of such a supersolid phase, which has been sought experimentally for over fifty years. In 2004, E. Kim and M. W. Chan claimed to have finally succeeded in observing supersolid helium [link], but it seems fair to state that their claim is not universally accepted (due to the active debate about this possible phase) at the time of this writing.

Other scenarios of SF in the presence of localization, such as in disordered systems, or in confined geometries, have been the subject of much experimental and theoretical investigation over the past two decades. Besides its unquestionable fundamental importance, the subject of SF in disorder or confinement is of interest due to the connection of SF to superconductivity [link], whose potential technological impact can hardly be overstated. Superconductivity occurs in crystals, i.e., systems which are inevitably “dirty,” disordered by defects, impurities and so on.

This thesis is a contribution to the theoretical understanding of SF in condensed matter systems where particles are subjected to localization. We focus our attention on three specific scenarios, namely crystallization, occurring as a result of interactions among particles, induced by an external potential, and disorder. As an archetypal model of a superfluid, we adopted the lattice hardcore Bose model, with the addition of nearest-neighbour and next-nearest-neighbour interactions.
The choice of this model is motivated essentially by its simplicity. It is conceptually related to the Bose Hubbard model (BHM) [link], which is a minimal model of SF extensively used to gain insight into fundamental properties of the superfluid phase. The purpose of utilizing such a model in a theoretical study is generally not that of obtaining precise, quantitatively reliable theoretical predictions applicable to an actual experimental system, but rather that of determining broad conditions under which SF can manifest itself, possibly in concomitance with other types of order.

Over the past decade, however, this state of affairs has changed somewhat, as simple models such as the BHM can actually be regarded as realistic descriptions of assemblies of cold atoms trapped in optical lattices [link], [link], [link]. Thus, models regarded until over a decade ago as being of “academic interest” only, are now eliciting renewed attention, as potentially allowing for quantitative predictions for experimental systems realizable in the laboratory.

Our studies consist of state-of-the-art numerical simulations based on the Worm Algorithm (WA) [link], [link]. This computational tool belongs to the class of Quantum Monte Carlo (QMC) techniques, which are widely regarded as the method of choice to investigate equilibrium thermodynamic properties of Bose systems at finite temperature. The advantage of this methodology is that it allows one to obtain reliable numerical results, virtually exact and free from approximations, for a wide class of Bose systems, featuring very different interactions; it also provides direct access to relevant physical quantities that are used to characterize experimentally the superfluid phase of matter.

**Summary of original research**

In the course of this investigation, we carried out three separate, but conceptually related projects, namely:

**Vacancy-based supersolidity**

In this part of our research, we investigated the scenario of vacancy-based supersolidity near crystallization induced by interparticle interactions. This is the original proposal for a supersolid phase [link], [link], namely one displaying simultaneously crystalline order and superfluidity. In their scenarios, Andreev and Lifshitz hypothesized that point defects such as vacancies (where the particles are removed from the lattice sites) could enjoy high mobility, hop from one lattice site to an adjacent one, and essentially act as a weakly interacting dilute lattice Bose gas, which could undergo Bose condensation and enjoy frictionless flow. However, recent first-principles numerical simulations have yielded strong evidence that such a scenario does not actually occur in solid helium, as any dilute gas of weakly interacting vacancies would be unstable against separation of the system into two phases, one vacancy-free and the other rich in vacancies, which can then be removed from the system via an adjustment of the lattice constant [link].
Computational studies of lattice models such as the hardcore Bose one, have yielded evidence of supersolid phases for various lattice geometries [link], [link], [link], [link], [link], [link], [link], [link], [link]. In all such studies, however, the supersolid phase is based on *interstitials* (where the particles are inserted between lattice sites) rather than vacancies, i.e., there appears to be an asymmetry in the behaviour of such point defects. In contrast, doping with vacancies results in the coexistence of an insulating crystal and a superfluid by the formation of a domain wall (e.g., a line of hole separated particles into two parts) [link]. Our study was aimed at elucidating the asymmetry between the behaviour of vacancies and interstitials.

We carried out calculations in the context of the hardcore Boson model on the square lattice, supplemented by nearest-neighbour and next-nearest-neighbour repulsive interactions. Our main results show that a vacancy-based supersolid phase is possible, and we obtained a simple criterion to predict its occurrence. We also studied the possible occurrence of a *commensurate* supersolid phase, namely one with neither vacancies nor interstitials, and found that no such phase exists, in accord with most other studies.

This research was published in


**Supersolidity in a periodic superlattice**

In this project, we considered another scenario of supersolidity near crystallization, this time not arising spontaneously but rather induced by an external potential. This study is qualitatively relevant to helium films adsorbed on graphite.

In this work, we investigated theoretically the possible existence of supersolid behaviour near a crystalline phase stabilized by an external periodic potential, which plays the same role as the adsorption sites of a corrugated substrate. Such a crystalline phase is *not* present in the phase diagram of the system in the absence of an external potential.

The purpose of this study is to provide a simple theoretical framework to interpret experimental studies probing for possible (commensurate) unconventional superfluid phases (or supersolid phases) of helium films on graphite. Our main finding is that these phases exist on both the interstitial and on the vacancy side of a commensurate (registered) crystal. A second, important conclusion of this study, is that the superfluid density *always vanishes* as the particle density hits a value corresponding to either a commensurate or incommensurate crystal. In this sense, the pinning potential does not give rise to fundamentally new behaviour, with respect to what is observed in this model near and/or at incommensurate crystal phases, in the absence of any external potential. The vanishing of the superfluid response at crystal density appears therefore to be a
general hallmark of any phase labelled as “supersolid”, occurring in a system of this type, i.e., in the presence of an external pinning potential.

This research was published in:


**Disorder-induced superfluidity**

In this study, we investigated a different scenario of unconventional superfluidity, arising from the presence of disorder.

We consider a model of lattice hardcore bosons with a strong *attractive* nearest-neighbour interaction. Such a system does *not* display a superfluid phase in the absence of disorder.

We present here strong numerical evidence for disorder-induced superfluidity. Specifically, we show that at low temperature and in a small range of attractive interactions, disorder of sufficient strength stabilizes a “glassy” superfluid phase or *superglass*. Aside from supersolid $^4$He, such a scenario is possibly relevant to other condensed matter systems, e.g., high-temperature superconductors [link], as well as to the elusive superfluid phase of bulk molecular hydrogen [link], and to the role of substrate disorder in the superfluidity of (sub)monolayer helium films [link], [link].

This research was published in:


**Thesis outline**

This thesis is organized as follows: in Chapter 2, we introduce the model of interacting bosons that is common to all the projects carried out in this work. Next, we describe the computational methodology in Chapter 3. In Chapter 4, we will discuss the results of our research efforts, and Chapter 5 summarizes the thesis.