



Null and Alternative Hypotheses

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The actual test begins by considering two hypotheses. They are called the null hypothesis and the alternative hypothesis. These hypotheses contain opposing viewpoints.

H_0 : The null hypothesis: It is a statement about the population that either is believed to be true or is used to put forth an argument unless it can be shown to be incorrect beyond a reasonable doubt.

H_a : The alternative hypothesis: It is a claim about the population that is contradictory to H_0 and what we conclude when we reject H_0 .

Since the null and alternative hypotheses are contradictory, you must examine evidence to decide if you have enough evidence to reject the null hypothesis or not. The evidence is in the form of sample data.

After you have determined which hypothesis the sample supports, you make a **decision**. There are two options for a decision. They are "reject H_0 " if the sample information favors the alternative hypothesis or "do not reject H_0 " or "decline to reject H_0 " if the sample information is insufficient to reject the null hypothesis.

Mathematical Symbols Used in H_0 and H_a :

H_0	H_a
equal (=)	not equal (\neq) or greater than ($>$) or less than ($<$)
greater than or equal to (\geq)	less than ($<$)
less than or equal to (\leq)	more than ($>$)

Note

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H_0 always has a symbol with an equal in it. H_a never has a symbol with an equal in it. The choice of symbol depends on the wording of the hypothesis test. However, be aware that many researchers (including one of the co-authors in research work) use = in the null hypothesis, even with > or < as the symbol in the alternative hypothesis. This practice is acceptable because we only make the decision to reject or not reject the null hypothesis.

H_0 : No more than 30% of the registered voters in Santa Clara County voted in the primary election. $p \leq 30$

H_a : More than 30% of the registered voters in Santa Clara County voted in the primary election. $p > 30$

Try It

A medical trial is conducted to test whether or not a new medicine reduces cholesterol by 25%. State the null and alternative hypotheses.

H_0 : The drug reduces cholesterol by 25%. $p = 0.25$

H_a : The drug does not reduce cholesterol by 25%. $p \neq 0.25$

We want to test whether the mean GPA of students in American colleges is different from 2.0 (out of 4.0). The null and alternative hypotheses are:

$H_0: \mu = 2.0$

$H_a: \mu \neq 2.0$

Try It

We want to test whether the mean height of eighth graders is 66 inches. State the null and alternative hypotheses. Fill in the correct symbol (=, \neq , \geq , $<$, \leq , $>$) for the null and alternative hypotheses.

1. $H_0: \mu _ 66$

2. $H_a: \mu _ 66$

1. $H_0 : \mu = 66$

2. $H_a : \mu \neq 66$

We want to test if college students take less than five years to graduate from college, on the average. The null and alternative hypotheses are:

$H_0: \mu \geq 5$

$H_a: \mu < 5$

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Try It

We want to test if it takes fewer than 45 minutes to teach a lesson plan. State the null and alternative hypotheses. Fill in the correct symbol ($=$, \neq , \geq , $<$, \leq , $>$) for the null and alternative hypotheses.

1. $H_0: \mu _ 45$
2. $H_a: \mu _ 45$

1. $H_0: \mu \geq 45$
2. $H_a: \mu < 45$

In an issue of *U. S. News and World Report*, an article on school standards stated that about half of all students in France, Germany, and Israel take advanced placement exams and a third pass. The same article stated that 6.6% of U.S. students take advanced placement exams and 4.4% pass. Test if the percentage of U.S. students who take advanced placement exams is more than 6.6%. State the null and alternative hypotheses.

$$H_0: p \leq 0.066$$

$$H_a: p > 0.066$$

Try It

On a state driver's test, about 40% pass the test on the first try. We want to test if more than 40% pass on the first try. Fill in the correct symbol ($=$, \neq , \geq , $<$, \leq , $>$) for the null and alternative hypotheses.

1. $H_0: p _ 0.40$
2. $H_a: p _ 0.40$

1. $H_0: p = 0.40$
2. $H_a: p > 0.40$

Collaborative Exercise

Bring to class a newspaper, some news magazines, and some Internet articles . In groups, find articles from which your group can write null and alternative hypotheses. Discuss your hypotheses with the rest of the class.

Chapter Review

In a **hypothesis test**, sample data is evaluated in order to arrive at a decision about some type of claim. If certain conditions about the sample are satisfied, then the claim can be evaluated for a population. In a hypothesis test, we:

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1. Evaluate the **null hypothesis**, typically denoted with H_0 . The null is not rejected unless the hypothesis test shows otherwise. The null statement must always contain some form of equality ($=$, \leq or \geq)
2. Always write the **alternative hypothesis**, typically denoted with H_a or H_1 , using less than, greater than, or not equals symbols, i.e., (\neq , $>$, or $<$).
3. If we reject the null hypothesis, then we can assume there is enough evidence to support the alternative hypothesis.
4. Never state that a claim is proven true or false. Keep in mind the underlying fact that hypothesis testing is based on probability laws; therefore, we can talk only in terms of non-absolute certainties.

Formula Review

H_0 and H_a are contradictory.

If H_0 has:	equal ($=$)	greater than or equal to (\geq)	less than or equal to (\leq)
then H_a has:	not equal (\neq) or greater than ($>$) or less than ($<$)	less than ($<$)	greater than ($>$)

If $\alpha \leq p$ -value, then do not reject H_0 .

If $\alpha > p$ -value, then reject H_0 .

α is preconceived. Its value is set before the hypothesis test starts. The p -value is calculated from the data.

You are testing that the mean speed of your cable Internet connection is more than three Megabits per second. What is the random variable? Describe in words.

The random variable is the mean Internet speed in Megabits per second.

You are testing that the mean speed of your cable Internet connection is more than three Megabits per second. State the null and alternative hypotheses.

The American family has an average of two children. What is the random variable? Describe in words.

The random variable is the mean number of children an American family has.

The mean entry level salary of an employee at a company is \$58,000. You believe it is higher for IT professionals in the company. State the null and alternative hypotheses.

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A sociologist claims the probability that a person picked at random in Times Square in New York City is visiting the area is 0.83. You want to test to see if the proportion is actually less. What is the random variable? Describe in words.

The random variable is the proportion of people picked at random in Times Square visiting the city.

A sociologist claims the probability that a person picked at random in Times Square in New York City is visiting the area is 0.83. You want to test to see if the claim is correct. State the null and alternative hypotheses.

In a population of fish, approximately 42% are female. A test is conducted to see if, in fact, the proportion is less. State the null and alternative hypotheses.

1. $H_0: p = 0.42$
2. $H_a: p < 0.42$

Suppose that a recent article stated that the mean time spent in jail by a first-time convicted burglar is 2.5 years. A study was then done to see if the mean time has increased in the new century. A random sample of 26 first-time convicted burglars in a recent year was picked. The mean length of time in jail from the survey was 3 years with a standard deviation of 1.8 years. Suppose that it is somehow known that the population standard deviation is 1.5. If you were conducting a hypothesis test to determine if the mean length of jail time has increased, what would the null and alternative hypotheses be? The distribution of the population is normal.

1. $H_0: \underline{\hspace{2cm}}$
2. $H_a: \underline{\hspace{2cm}}$

A random survey of 75 death row inmates revealed that the mean length of time on death row is 17.4 years with a standard deviation of 6.3 years. If you were conducting a hypothesis test to determine if the population mean time on death row could likely be 15 years, what would the null and alternative hypotheses be?

1. $H_0: \underline{\hspace{2cm}}$
 2. $H_a: \underline{\hspace{2cm}}$
1. $H_0: \mu = 15$
 2. $H_a: \mu \neq 15$

The National Institute of Mental Health published an article stating that in any one-year period, approximately 9.5 percent of American adults suffer from depression or a depressive illness. Suppose that in a survey of 100 people in a certain town, seven of them suffered from depression or a depressive illness. If you were conducting a

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hypothesis test to determine if the true proportion of people in that town suffering from depression or a depressive illness is lower than the percent in the general adult American population, what would the null and alternative hypotheses be?

1. H_0 : _____
2. H_a : _____

Homework

Some of the following statements refer to the null hypothesis, some to the alternate hypothesis.

State the null hypothesis, H_0 , and the alternative hypothesis, H_a , in terms of the appropriate parameter (μ or p).

1. The mean number of years Americans work before retiring is 34.
2. At most 60% of Americans vote in presidential elections.
3. The mean starting salary for San Jose State University graduates is at least \$100,000 per year.
4. Twenty-nine percent of high school seniors get drunk each month.
5. Fewer than 5% of adults ride the bus to work in Los Angeles.
6. The mean number of cars a person owns in her lifetime is not more than ten.
7. About half of Americans prefer to live away from cities, given the choice.
8. Europeans have a mean paid vacation each year of six weeks.
9. The chance of developing breast cancer is under 11% for women.
10. Private universities' mean tuition cost is more than \$20,000 per year.
 1. $H_0: \mu = 34; H_a: \mu \neq 34$
 2. $H_0: p \leq 0.60; H_a: p > 0.60$
 3. $H_0: \mu \geq 100,000; H_a: \mu < 100,000$
 4. $H_0: p = 0.29; H_a: p \neq 0.29$
 5. $H_0: p = 0.05; H_a: p < 0.05$
 6. $H_0: \mu \leq 10; H_a: \mu > 10$
 7. $H_0: p = 0.50; H_a: p \neq 0.50$
 8. $H_0: \mu = 6; H_a: \mu \neq 6$
 9. $H_0: p \geq 0.11; H_a: p < 0.11$
 10. $H_0: \mu \leq 20,000; H_a: \mu > 20,000$

Over the past few decades, public health officials have examined the link between weight concerns and teen girls' smoking. Researchers surveyed a group of 273 randomly selected teen girls living in Massachusetts (between 12 and 15 years old). After four years the girls were surveyed again. Sixty-three said they smoked to stay thin. Is there

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good evidence that more than thirty percent of the teen girls smoke to stay thin? The alternative hypothesis is:

1. $p < 0.30$
2. $p \leq 0.30$
3. $p \geq 0.30$
4. $p > 0.30$

A statistics instructor believes that fewer than 20% of Evergreen Valley College (EVC) students attended the opening night midnight showing of the latest Harry Potter movie. She surveys 84 of her students and finds that 11 attended the midnight showing. An appropriate alternative hypothesis is:

1. $p = 0.20$
2. $p > 0.20$
3. $p < 0.20$
4. $p \leq 0.20$

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Previously, an organization reported that teenagers spent 4.5 hours per week, on average, on the phone. The organization thinks that, currently, the mean is higher. Fifteen randomly chosen teenagers were asked how many hours per week they spend on the phone. The sample mean was 4.75 hours with a sample standard deviation of 2.0. Conduct a hypothesis test. The null and alternative hypotheses are:

1. $H_0: \bar{x} = 4.5, H_a: \bar{x} > 4.5$
2. $H_0: \mu \geq 4.5, H_a: \mu < 4.5$
3. $H_0: \mu = 4.75, H_a: \mu > 4.75$
4. $H_0: \mu = 4.5, H_a: \mu > 4.5$

References

Data from the National Institute of Mental Health. Available online at <http://www.nimh.nih.gov/publicat/depression.cfm>.