



One-Way ANOVA

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The purpose of a one-way ANOVA test is to determine the existence of a statistically significant difference among several group means. The test actually uses variances to help determine if the means are equal or not. In order to perform a one-way ANOVA test, there are five basic **assumptions** to be fulfilled:

1. Each population from which a sample is taken is assumed to be normal.
2. All samples are randomly selected and independent.
3. The populations are assumed to have equal standard deviations (**or variances**).
4. The factor is a categorical variable.
5. The response is a numerical variable.

The Null and Alternative Hypotheses

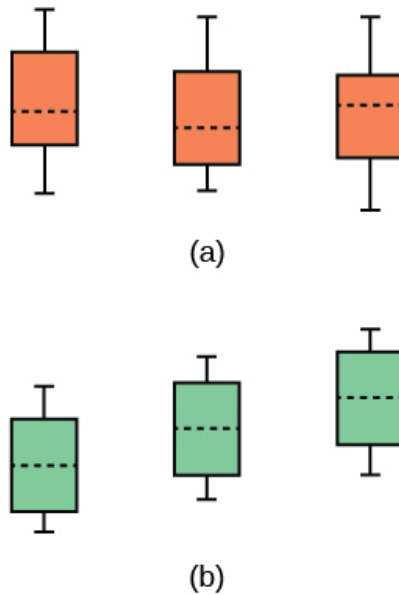
The null hypothesis is simply that all the group population means are the same. The alternative hypothesis is that at least one pair of means is different. For example, if there are k groups:

$$H_0: \mu_1 = \mu_2 = \mu_3 = \dots = \mu_k$$

H_a : At least two of the group means $\mu_1, \mu_2, \mu_3, \dots, \mu_k$ are not equal.

The graphs, a set of box plots representing the distribution of values with the group means indicated by a horizontal line through the box, help in the understanding of the hypothesis test. In the first graph (red box plots), $H_0: \mu_1 = \mu_2 = \mu_3$ and the three populations have the same distribution if the null hypothesis is true. The variance of the combined data is approximately the same as the variance of each of the populations.

If the null hypothesis is false, then the variance of the combined data is larger which is caused by the different means as shown in the second graph (green box plots).



(a) H_0 is true. All means are the same; the differences are due to random variation. (b) H_0 is not true. All means are not the same; the differences are too large to be due to random variation.

Chapter Review

Analysis of variance extends the comparison of two groups to several, each a level of a categorical variable (factor). Samples from each group are independent, and must be randomly selected from normal populations with equal variances. We test the null hypothesis of equal means of the response in every group versus the alternative hypothesis of one or more group means being different from the others. A one-way ANOVA hypothesis test determines if several population means are equal. The distribution for the test is the F distribution with two different degrees of freedom.

Assumptions:

1. Each population from which a sample is taken is assumed to be normal.
2. All samples are randomly selected and independent.
3. The populations are assumed to have equal standard deviations (or variances).

Use the following information to answer the next five exercises. There are five basic assumptions that must be fulfilled in order to perform a one-way ANOVA test. What are they?

Write one assumption.

Each population from which a sample is taken is assumed to be normal.

Write another assumption.

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Write a third assumption.

The populations are assumed to have equal standard deviations (or variances).

Write a fourth assumption.

Write the final assumption.

The response is a numerical value.

State the null hypothesis for a one-way ANOVA test if there are four groups.

State the alternative hypothesis for a one-way ANOVA test if there are three groups.

H_a : At least two of the group means μ_1, μ_2, μ_3 are not equal.

When do you use an ANOVA test?

Homework

Three different traffic routes are tested for mean driving time. The entries in the table are the driving times in minutes on the three different routes. The one-way ANOVA results are shown in [\[link\]](#).

Route 1	Route 2	Route 3
30	27	16
32	29	41
27	28	22
35	36	31

State SS_{between} , SS_{within} , and the F statistic.

$$SS_{\text{between}} = 26$$

$$SS_{\text{within}} = 441$$

$$F = 0.2653$$

Suppose a group is interested in determining whether teenagers obtain their drivers licenses at approximately the same average age across the country. Suppose that the following data are randomly collected from five teenagers in each region of the country. The numbers represent the age at which teenagers obtained their drivers licenses.

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	Northeast	South	West	Central	East
	16.3	16.9	16.4	16.2	17.1
	16.1	16.5	16.5	16.6	17.2
	16.4	16.4	16.6	16.5	16.6
	16.5	16.2	16.1	16.4	16.8
$\bar{x} =$	_____	_____	_____	_____	_____
$s^2 =$	_____	_____	_____	_____	_____

State the hypotheses.

H_0 : _____

H_a : _____