



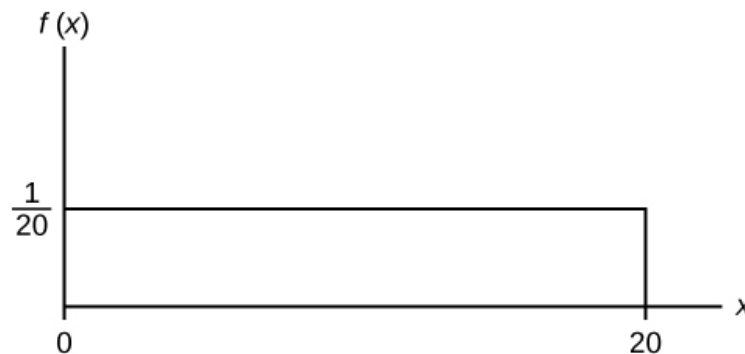
Continuous Probability Functions

By:

OpenStaxCollege

We begin by defining a continuous probability density function. We use the function notation $f(x)$. Intermediate algebra may have been your first formal introduction to functions. In the study of probability, the functions we study are special. We define the function $f(x)$ so that the area between it and the x -axis is equal to a probability. Since the maximum probability is one, the maximum area is also one. **For continuous probability distributions, PROBABILITY = AREA.**

Consider the function $f(x) = \frac{1}{20}$ for $0 \leq x \leq 20$. $x =$ a real number. The graph of $f(x) = \frac{1}{20}$ is a horizontal line. However, since $0 \leq x \leq 20$, $f(x)$ is restricted to the portion between $x = 0$ and $x = 20$, inclusive.



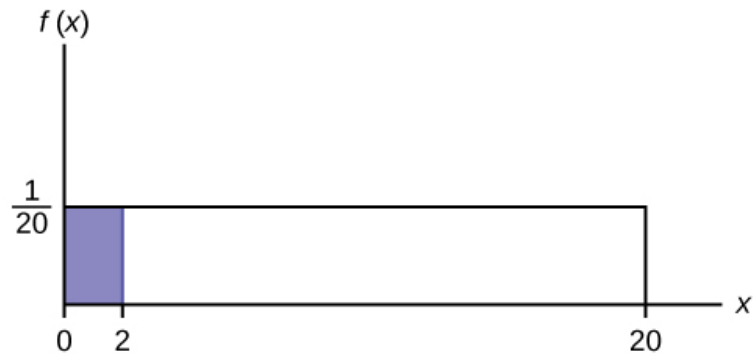
$$f(x) = \frac{1}{20} \text{ for } 0 \leq x \leq 20.$$

The graph of $f(x) = \frac{1}{20}$ is a horizontal line segment when $0 \leq x \leq 20$.

The area between $f(x) = \frac{1}{20}$ where $0 \leq x \leq 20$ and the x -axis is the area of a rectangle with base = 20 and height = $\frac{1}{20}$.

$$\text{AREA} = 20\left(\frac{1}{20}\right) = 1$$

Suppose we want to find the area between $f(x) = \frac{1}{20}$ and the x -axis where $0 < x < 2$.



$$\text{AREA} = (2 - 0)\left(\frac{1}{20}\right) = 0.1$$

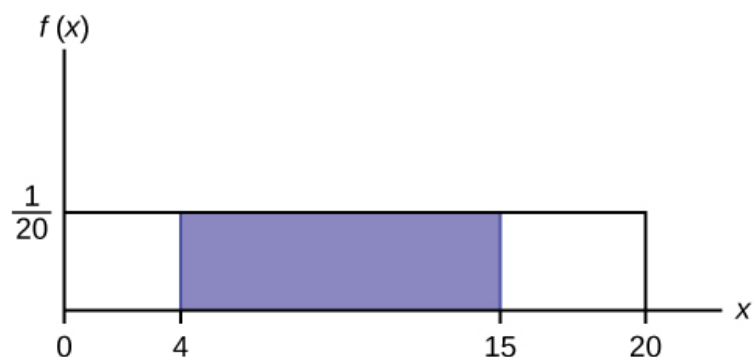
$$(2 - 0) = 2 = \text{base of a rectangle}$$

Reminder

area of a rectangle = (base)(height).

The area corresponds to a probability. The probability that x is between zero and two is 0.1, which can be written mathematically as $P(0 < x < 2) = P(x < 2) = 0.1$.

Suppose we want to find the area between $f(x) = \frac{1}{20}$ and the x -axis where $4 < x < 15$.



$$\text{AREA} = (15 - 4)\left(\frac{1}{20}\right) = 0.55$$

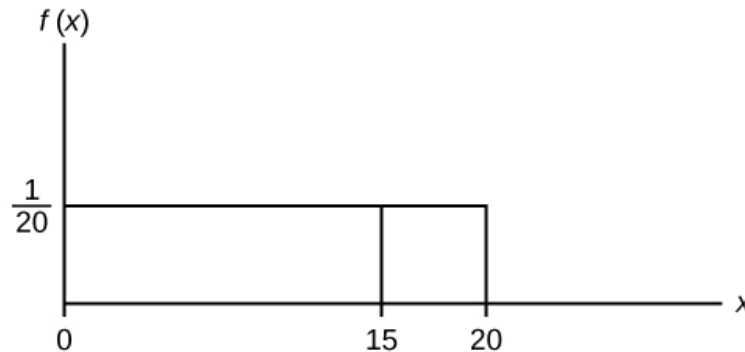
$$\text{AREA} = (15 - 4)\left(\frac{1}{20}\right) = 0.55$$

Continuous Probability Functions

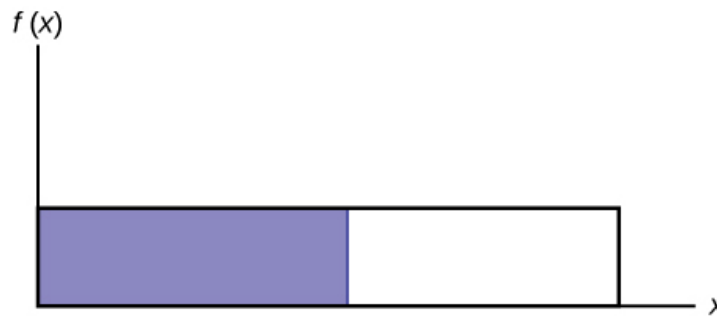
$$(15 - 4) = 11 = \text{the base of a rectangle}$$

The area corresponds to the probability $P(4 < x < 15) = 0.55$.

Suppose we want to find $P(x = 15)$. On an x-y graph, $x = 15$ is a vertical line. A vertical line has no width (or zero width). Therefore, $P(x = 15) = (\text{base})(\text{height}) = (0)\left(\frac{1}{20}\right) = 0$

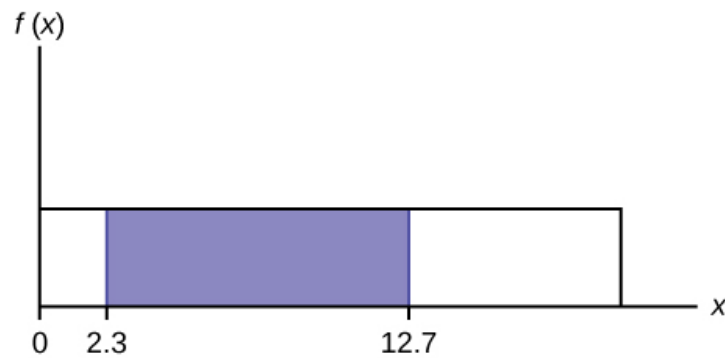


$P(X \leq x)$ (can be written as $P(X < x)$ for continuous distributions) is called the cumulative distribution function or CDF. Notice the "less than or equal to" symbol. We can use the CDF to calculate $P(X > x)$. The CDF gives "area to the left" and $P(X > x)$ gives "area to the right." We calculate $P(X > x)$ for continuous distributions as follows: $P(X > x) = 1 - P(X < x)$.



Label the graph with $f(x)$ and x . Scale the x and y axes with the maximum x and y values.
 $f(x) = \frac{1}{20}, 0 \leq x \leq 20$.

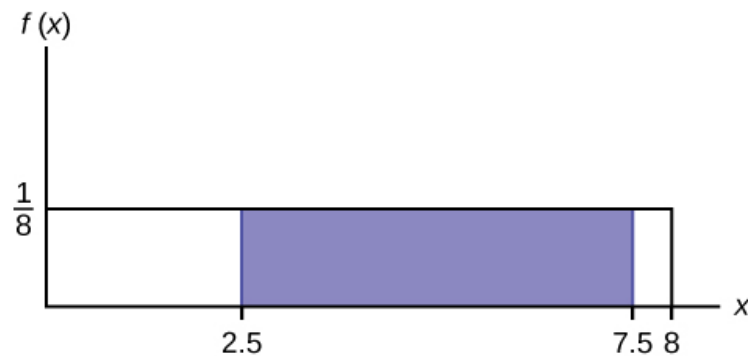
To calculate the probability that x is between two values, look at the following graph. Shade the region between $x = 2.3$ and $x = 12.7$. Then calculate the shaded area of a rectangle.



$$P(2.3 < x < 12.7) = (\text{base})(\text{height}) = (12.7 - 2.3)\left(\frac{1}{20}\right) = 0.52$$

Try It

Consider the function $f(x) = \frac{1}{8}$ for $0 \leq x \leq 8$. Draw the graph of $f(x)$ and find $P(2.5 < x < 7.5)$.

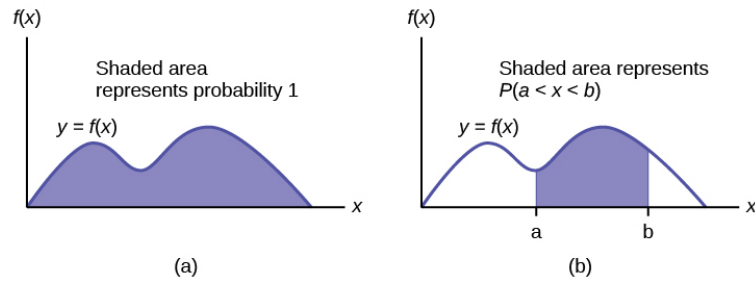


$$P(2.5 < x < 7.5) = 0.625$$

Chapter Review

The probability density function (pdf) is used to describe probabilities for continuous random variables. The area under the density curve between two points corresponds to the probability that the variable falls between those two values. In other words, the area under the density curve between points a and b is equal to $P(a < x < b)$. The cumulative distribution function (cdf) gives the probability as an area. If X is a continuous random variable, the probability density function (pdf), $f(x)$, is used to draw the graph of the probability distribution. The total area under the graph of $f(x)$ is one. The area under the graph of $f(x)$ and between values a and b gives the probability $P(a < x < b)$.

Continuous Probability Functions



The cumulative distribution function (cdf) of X is defined by $P(X \leq x)$. It is a function of x that gives the probability that the random variable is less than or equal to x .

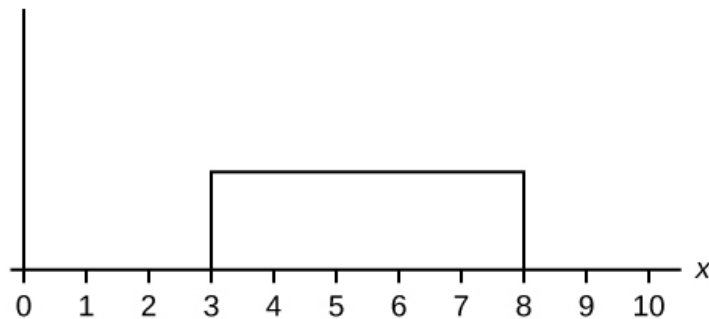
Formula Review

Probability density function (pdf) $f(x)$:

- $f(x) \geq 0$
- The total area under the curve $f(x)$ is one.

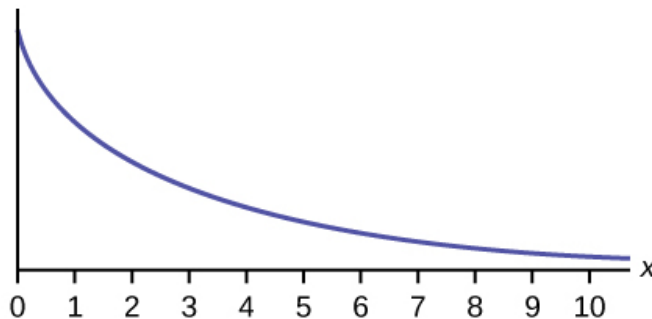
Cumulative distribution function (cdf): $P(X \leq x)$

Which type of distribution does the graph illustrate?



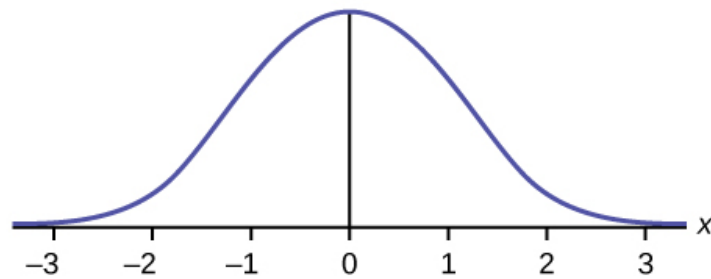
Uniform Distribution

Which type of distribution does the graph illustrate?



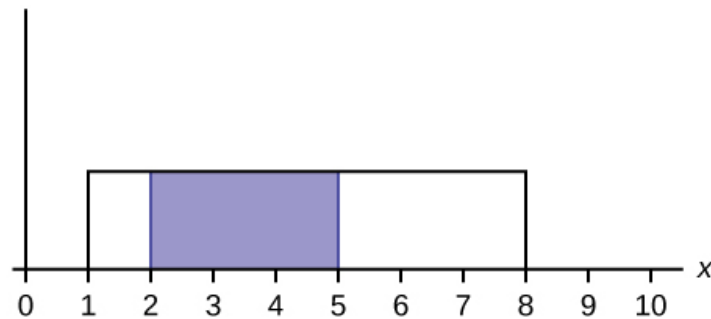
Continuous Probability Functions

Which type of distribution does the graph illustrate?

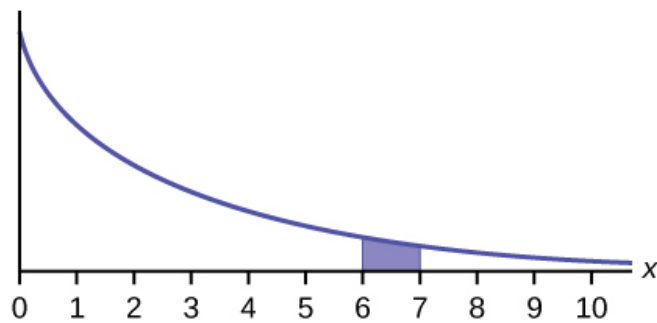


Normal Distribution

What does the shaded area represent? $P(__ < x < __)$



What does the shaded area represent? $P(__ < x < __)$



$$P(6 < x < 7)$$

For a continuous probability distribution, $0 \leq x \leq 15$. What is $P(x > 15)$?

What is the area under $f(x)$ if the function is a continuous probability density function?

one

For a continuous probability distribution, $0 \leq x \leq 10$. What is $P(x = 7)$?

Continuous Probability Functions

A **continuous** probability function is restricted to the portion between $x = 0$ and 7. What is $P(x = 10)$?

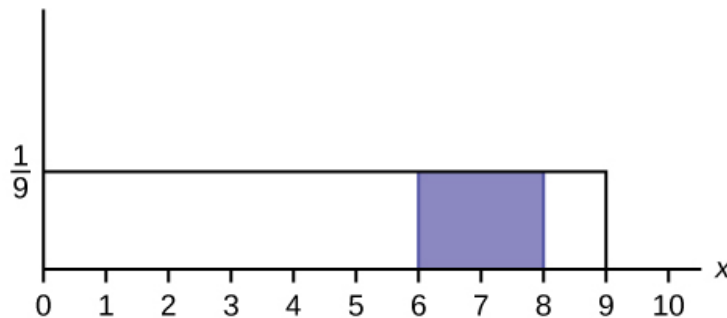
zero

$f(x)$ for a continuous probability function is $\frac{1}{5}$, and the function is restricted to $0 \leq x \leq 5$. What is $P(x < 0)$?

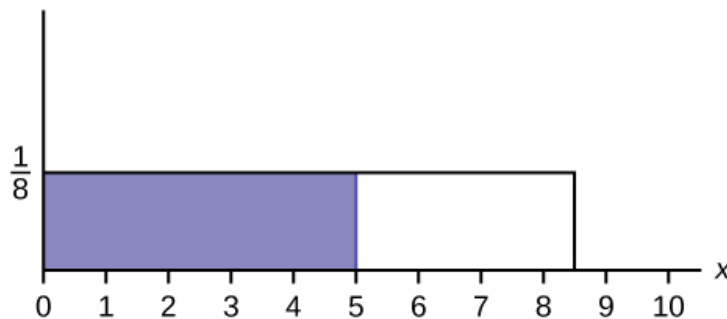
$f(x)$, a continuous probability function, is equal to $\frac{1}{12}$, and the function is restricted to $0 \leq x \leq 12$. What is $P(0 < x < 12)$?

one

Find the probability that x falls in the shaded area.

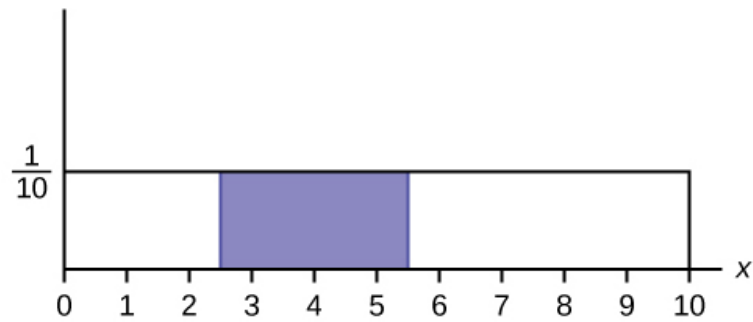


Find the probability that x falls in the shaded area.



0.625

Find the probability that x falls in the shaded area.



$f(x)$, a continuous probability function, is equal to $\frac{1}{3}$ and the function is restricted to $1 \leq x \leq 4$. Describe $P(x > \frac{3}{2})$.

The probability is equal to the area from $x = \frac{3}{2}$ to $x = 4$ above the x-axis and up to $f(x) = \frac{1}{3}$.

Homework

For each probability and percentile problem, draw the picture.

Consider the following experiment. You are one of 100 people enlisted to take part in a study to determine the percent of nurses in America with an R.N. (registered nurse) degree. You ask nurses if they have an R.N. degree. The nurses answer “yes” or “no.” You then calculate the percentage of nurses with an R.N. degree. You give that percentage to your supervisor.

1. What part of the experiment will yield discrete data?
2. What part of the experiment will yield continuous data?

When age is rounded to the nearest year, do the data stay continuous, or do they become discrete? Why?

Age is a measurement, regardless of the accuracy used.