



Facts About the Chi-Square Distribution

By:
OpenStaxCollege

The notation for the chi-square distribution is:

$$\chi \sim \chi_{df}^2$$

where df = degrees of freedom which depends on how chi-square is being used. (If you want to practice calculating chi-square probabilities then use $df = n - 1$. The degrees of freedom for the three major uses are each calculated differently.)

For the χ^2 distribution, the population mean is $\mu = df$ and the population standard deviation is $\sigma = \sqrt{2(df)}$.

The random variable is shown as χ^2 , but may be any upper case letter.

The random variable for a chi-square distribution with k degrees of freedom is the sum of k independent, squared standard normal variables.

$$\chi^2 = (Z_1)^2 + (Z_2)^2 + \dots + (Z_k)^2$$

1. The curve is nonsymmetrical and skewed to the right.
2. There is a different chi-square curve for each df .
3. The test statistic for any test is always greater than or equal to zero.
4. When $df > 90$, the chi-square curve approximates the normal distribution. For $X \sim \chi_{1,000}^2$ the mean, $\mu = df = 1,000$ and the standard deviation, $\sigma = \sqrt{2(1,000)} = 44.7$. Therefore, $X \sim N(1,000, 44.7)$, approximately.
5. The mean, μ , is located just to the right of the peak.

References

Data from *Parade Magazine*.

“HIV/AIDS Epidemiology Santa Clara County.” Santa Clara County Public Health Department, May 2011.

Chapter Review

The chi-square distribution is a useful tool for assessment in a series of problem categories. These problem categories include primarily (i) whether a data set fits a particular distribution, (ii) whether the distributions of two populations are the same, (iii) whether two events might be independent, and (iv) whether there is a different variability than expected within a population.

An important parameter in a chi-square distribution is the degrees of freedom df in a given problem. The random variable in the chi-square distribution is the sum of squares of df standard normal variables, which must be independent. The key characteristics of the chi-square distribution also depend directly on the degrees of freedom.

The chi-square distribution curve is skewed to the right, and its shape depends on the degrees of freedom df . For $df > 90$, the curve approximates the normal distribution. Test statistics based on the chi-square distribution are always greater than or equal to zero. Such application tests are almost always right-tailed tests.

Formula Review

$\chi^2 = (Z_1)^2 + (Z_2)^2 + \dots + (Z_{df})^2$ chi-square distribution random variable

$\mu_{\chi^2} = df$ chi-square distribution population mean

$\sigma_{\chi^2} = \sqrt{2(df)}$ Chi-Square distribution population standard deviation

If the number of degrees of freedom for a chi-square distribution is 25, what is the population mean and standard deviation?

mean = 25 and standard deviation = 7.0711

If $df > 90$, the distribution is _____. If $df = 15$, the distribution is _____.

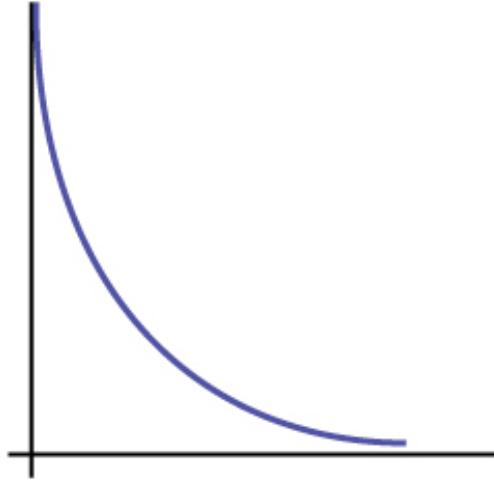
When does the chi-square curve approximate a normal distribution?

Facts About the Chi-Square Distribution

when the number of degrees of freedom is greater than 90

Where is μ located on a chi-square curve?

Is it more likely the df is 90, 20, or two in the graph?



$df = 2$

Homework

Decide whether the following statements are true or false.

As the number of degrees of freedom increases, the graph of the chi-square distribution looks more and more symmetrical.

true

The standard deviation of the chi-square distribution is twice the mean.

The mean and the median of the chi-square distribution are the same if $df = 24$.

false